

Neutron-Proton pairing revisited

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Abstract

We reexamine neutron-proton pairing as a phenomenon that should be explainable in a microscopic theory of nuclear binding energies. Empirically, there is an increased separation energy when both neutron and proton numbers are even or if they are both odd. The enhancement is present at some level in nearly all nuclei: the separation energy difference has the opposite sign in less than 1% of the cases in which sufficient data exist. We discuss the possible origin of the effect in the context of density functional theory (DFT) and its extensions. Neutron-proton pairing from the Hartree-Fock-Bogoliubov theory does not seem promising to explain the effect. We demonstrate that much of the increased binding in the odd-odd system might be understood as a recoupling energy. This suggests that the DFT should be extended by angular momentum projection to describe the effect.

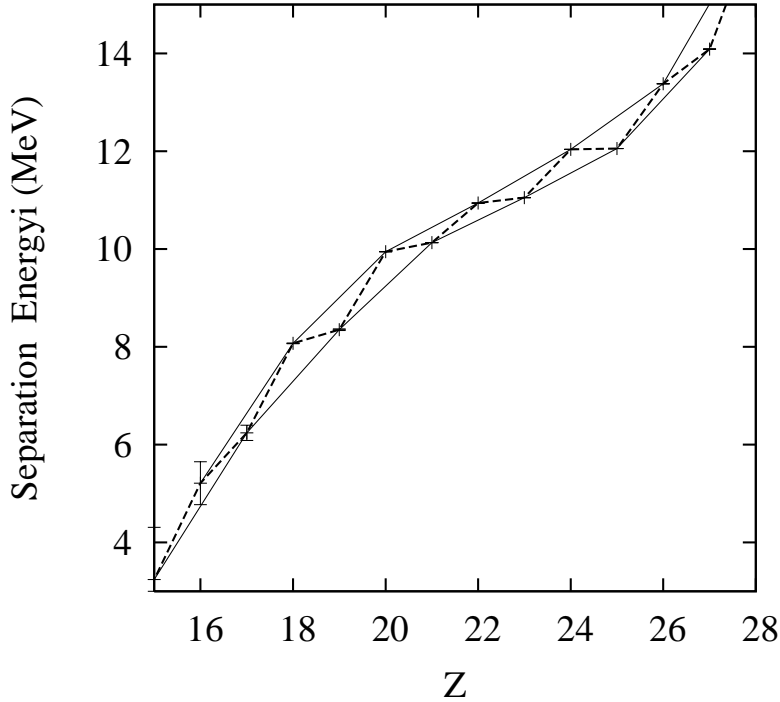


FIG. 1: Neutron-proton pairing effect as seen in the neutron separation energy for $N = 28$ as a function of proton number Z . There is a consistent offset of the separation energies of odd- Z nuclei as compared with the average of the neighboring even- Z nuclei.

It has been known for a long time that the nuclear binding has a mild dependence on the combined even-odd parities of proton and neutron numbers[1, p.171],[2, 3]. To see the effect, Fig. 1 shows the neutron separation energies for nuclei with neutron number $N = 28$ as a function of proton number Z ; the separation energy is expressed in terms of the binding energy as $S_n(N, Z) = B(N, Z) - B(N - 1, Z)$. One sees that the neutron separation energies for even Z are systematically larger than the average of the separation energies for the neighboring odd- Z nuclei. Similar behavior is found for proton separation energies S_p in chains of isotopes. In that case S_p is greater if the number of neutrons is even than when the number of neutrons is odd.

To study this behavior in more detail, we examine the separation energy differences S_{n2p}, S_{p2n} , defined as the difference between the separation energy and the average for the two neighboring nuclei. This is

$$S_{n2p} = S_n(N, Z) - (S_n(N, Z + 1) + S_n(N, Z - 1)) / 2 \quad (1)$$

$$S_{p2n} = S_p(N, Z) - ((S_p(N + 1, Z) + S_n(N - 1, Z)) / 2$$

for neutrons and protons, respectively. These measures were first introduced by Jensen et al. [4]. With our notation, the usual measure for ordinary pairing is given by (ref. [1, eq. 2-92,93])

$$2\Delta_n \equiv S_{n2n} = S_n(N, Z) - ((S_n(N + 1, Z) + S_n(N - 1, Z)) / 2$$

for the neutron gap, Δ_n , and similarly S_{p2p} gives the proton gap. Most earlier studies of neutron-proton pairing used different measures for the effect. In early fits of the measured binding energies [3, 5], the effect was parameterized as

$$\delta \sim \text{mod}(N, 2) \text{mod}(Z, 2) / A.$$

and attributed to an enhancement in the neutron-proton interaction. In ref. [6], the parameterization was changed to one have an approximate $A^{-2/3}$ dependence on nuclear mass number,

$$\delta = K \text{mod}(N, 2) \text{mod}(Z, 2) / A^{2/3}. \quad (2)$$

In ref. [7] a 9-point difference formula was proposed to describe a neutron-proton pairing energy. This is to be compared the 6-point difference formula we use in eq. (1). We also mention the shell-based mass fits of Zeldes[2], which invoke a shell-dependent term similar to δ .

We find that the signs of S_{n2p} and S_{p2n} are remarkably consistent across the nuclear mass table. Taking the data from the 2003 Audi-Wapstra mass tables[8], there are 1412 nuclei which have values of S_{n2p} that are significant, i.e., have magnitudes larger the accumulated error in the experimental binding energies needed to construct the difference. Of these only 10 nuclei had a sign for S_{n2p} opposite to that seen in Fig. 1. Of the 1448 measured proton separations S_{p2n} , only 9 had the opposite sign. The nuclei with significant values of S_{p2n} for proton separations are shown in Fig. 2, with the few opposite-sign cases shown as the black squares. The plot for neutron separations is very similar. There is concentration in the light nuclei near the $N = Z$ line, but no obvious pattern elsewhere.

We have also examined the dependence of the magnitude of the separation energy differences on the mass numbers, A , of the nuclei. There is a great deal of scatter as shown in Fig. 3, but the trend is consistent with an $A^{-2/3}$ dependence as in eq. (2). The heavy line shows a least squares fit to the data, $|S_{p2n}| = 7.3/A^{2/3}$ MeV. The values for $|S_{n2p}|$ display a very similar distribution.

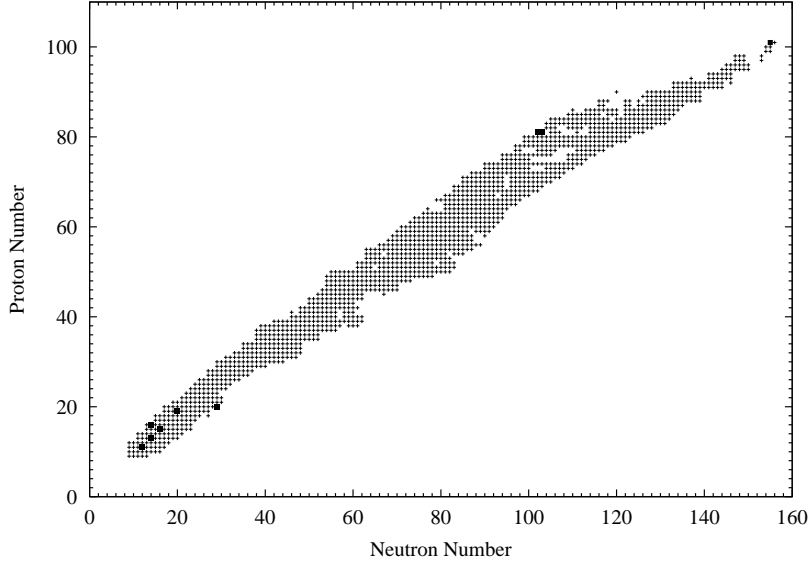


FIG. 2: Nuclei with measured proton separation energy differences S_{p2n} showing the cases (black squares) with opposite sign from the normal.

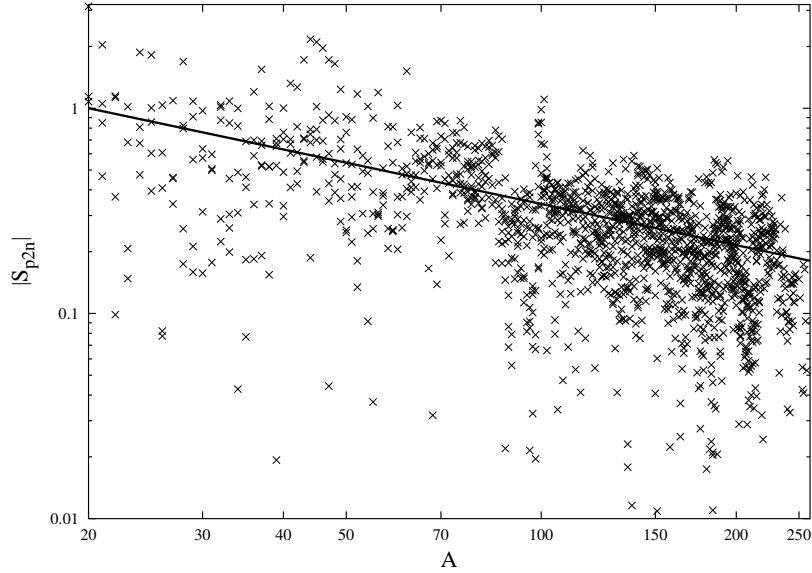


FIG. 3: $|S_{p2n}|$ as a function of A . The line shows the $A^{-2/3}$ fit eq. (2) with $K' = 7.3 \text{ MeV}$.

Finally, we plot in Fig. 4 all the measured nuclei, distinguishing by color those whose $|S_{p2n}|$ is larger or smaller than the average trend. We also find a very similar pattern for $|S_{n2p}|$. There is no visible dependence on shell closures.

We now turn to the question of what is responsible for the effect. The enhanced binding could arise by an increased attraction between an odd neutron and an odd proton. It

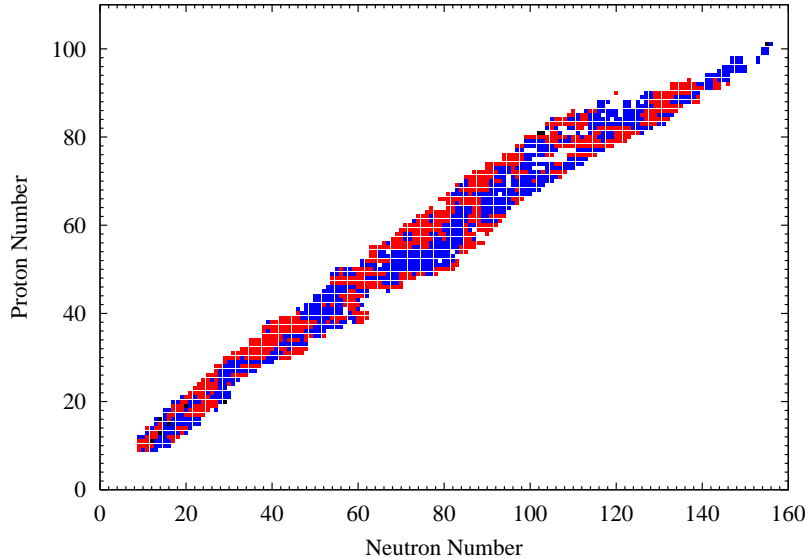


FIG. 4: Chart of nuclides showing the distribution of S_{p2n} values that are higher (red) or lower (blue) than the average trend $|S_{p2n}| = 7.3/A^{2/3}$ MeV.

could also arise by a mechanism that produced an increased binding in a nucleus with even numbers for both protons and neutrons. There is no way to distinguish these pictures by the observed systematics of the separation energy differences, because even-even and odd-odd binding energies contribute to the separation energy difference with the same weights. Both pictures are consistent with the strong similarity between the proton and neutron separation energy differences. Still, it is important to understand the origin of the effect if one is to construct accurate theories of nuclear binding based on microscopic theories such as the self-consistent mean field theory, also called density functional theory (DFT)[9, 10]. Ordinary pairing between like particles is quite well explained by the BCS or Hartree-Fock-Bogoliubov extension of DFT. This suggests generalizing the HFB theory to allow neutron-proton pairing. Certainly, at the $N = Z$ line neutron-proton pairing is on a same footing as like-particle pairing. Also, the neutron-proton interaction is stronger than the like-particle interaction, in free space. The special effects going on near $N = Z$ are often discussed as the “Wigner energy”. It is usually parameterized in a way that does not exhibit a neutron-proton pairing effect away from the $N = Z$ line and we shall consider it irrelevant to explain the effect. We note again that over half of the opposite-sign cases are near the $N = Z$ line. There have also been limited studies of neutron-proton pairing in the HFB theory [11, 12]. Typically, away from the $N = Z$ line, condensates form in the like-particle

sectors and prevent any pairing between neutrons and protons. We therefore doubt whether the effect can be explained without make some extension of the usual DFT+HFB theory.

There is a possible mechanism that only requires a mild extension of the DFT. That is to exploit the higher degeneracy of states in the odd-odd nucleus to recouple the neutron and proton more favorably. This is easiest to understand in the situations where the mean-field theory approaches either the spherical shell model or the strongly deformed limit. Indeed, Zeldes and Liran[2] may have had this mechanism in mind in their shell-based mass parameterization. For the shell-model limit, consider even-even nucleus (N,Z) that has a spherical mean field. An added neutron goes into a spherical shell j_n with an energy ϵ_{j_n} . Similarly, an added proton goes into a shell j_p . When there are both added neutrons and protons, there is an addition neutron-proton interaction energy $\langle j_n j_p | V_{np} | j_n j_p \rangle_J$ depending on the angular momentum of the pair J . The neutron separation energies for the nuclei with proton numbers $Z, Z + 1, Z + 2$ are, respectively,

$$S_n(N + 1, Z) = -\epsilon_{j_n}$$

$$S_n(N + 1, Z + 1) = -\epsilon_{j_n} - \langle j_n j_p | V_{np} | j_n j_p \rangle_{J_g}$$

$$S_n(N + 1, Z + 2) = -\epsilon_{j_n} - \langle j_n (j_p^2)^{J=0} | V_{np} | j_n (j_p^2)^{J=0} \rangle_{j_n}$$

In the second equation, J_g denotes the angular momentum of the odd-odd nucleus ground state. The last equation gives the neutron separation energy for the nucleus with two additional protons. Here the angular momentum coupling is determined by the three-particle wave function. In the spherical shell model, the two protons are coupled to angular momentum zero in the three-particle wave function $|j_n (j_p^2)^{J=0}\rangle$. Standard angular momentum recoupling gives the neutron-proton interaction as

$$\langle j_n (j_p^2)^{J=0} | V_{np} | j_n (j_p^2)^{J=0} \rangle_{j_n} = \sum_{J=|j_n-j_p|}^{j_n+j_p} (2J+1) \langle j_n j_p | V_{np} | j_n j_p \rangle_J / (2j_n+1)(2j_p+1)$$

Thus, in the shell model, the energy of the odd neutron when the proton number is even is the $(2J+1)$ -weighted average over the possible neutron-proton couplings.

This value can be estimated empirically from the spectrum of the odd-odd nuclei as the quantity

$$\delta_s = \sum_{J=|j_n-j_p|}^{j_n+j_p} (2J+1) E_J / (2j_n+1)(2j_p+1) \quad (3)$$

TABLE I: Comparison of neutron-proton pair interaction energies with the recoupling model, eq. 3. Energies are in MeV. The quantity δ_s is defined in eq. (3).

N	Z	S_{n2p}	S_{p2n}	δ_s
21	19	0.49	0.32	0.44
27	21	0.39	0.53	0.70
29	21	0.25	0.30	0.32
29	27	0.30	0.31	0.38
29	29	0.81	0.78	0.65
33	27	0.20	0.29	0.15
81	51	0.24	0.22	0.14
125	83	0.03	0.03	0.06
127	81		0.15	0.04
127	83	0.36	0.36	0.42

Here E_J are measured excitation energies of the levels of the multiplet in the odd-odd nucleus. The quantity δ_s is thus a measure of the enhancement of the neutron separation energy for an odd neutron in a nucleus with an odd number of protons.

For most odd-odd nuclei, the recoupling spectrum is difficult to determine due to the presence of other levels. However, near doubly magic nuclei it is often possible to make a spectroscopic identification[13]. Some cases where we could plausibly assign the members of the multiplet are shown in Table I. These results are also shown in Fig. 5. The recoupling energy δ_s has the same order of magnitude as the separation energy differences, and also varies from case to case in a similar way. However, there is considerable scatter leaving room for other mechanism to have a role.

If the odd-odd recoupling is dominant to produce the effect, it should be suppressed in deformed nuclei. The reason is that the coupling of the orbitals to the symmetry axis removes most of the degeneracy. Only the K quantum number remains, leaving a degeneracy between the two states $|K_p - K_n|$ and $K_p + K_n$ in the odd-odd nucleus. To see whether the suppression is indeed present, we have examined the separation energies differences for strongly deformed even-even nuclei. We took the classification of deformed nuclei from ref. [14], which used the theoretical criterion that the static deformation of the nucleus be

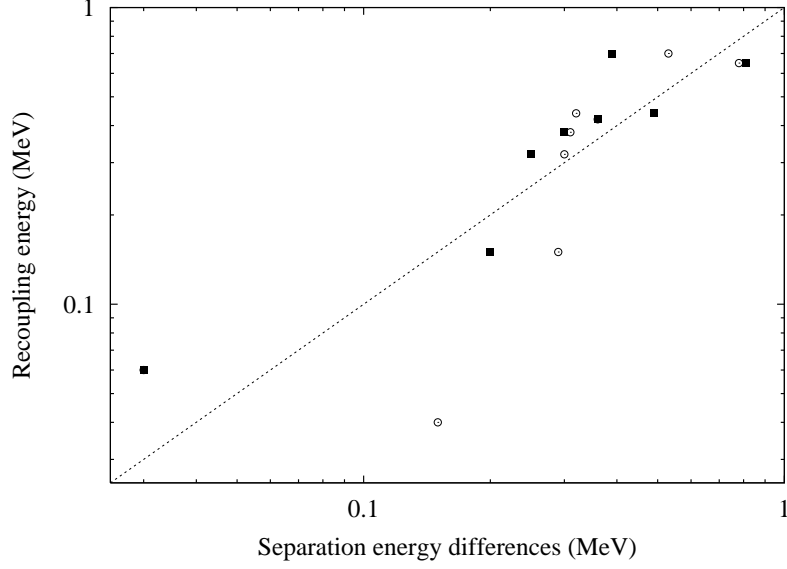


FIG. 5: Scatter plot of δ_s , eq. (2), compared with S_{n2p} (solid squares) and S_{p2n} (circles). The nuclei plotted are ^{40}K , ^{48}Sc , ^{50}Sc , ^{56}Co , ^{60}Co , ^{58}Cu , ^{208}Tl , ^{208}Bi , and ^{210}Bi .

larger than the fluctuations about the minimum. There are 92 nuclei with measured S_{n2p} that meet the criterion. Fitting eq. (2) to these nuclei, we find a slightly lower value for K , 5.7 MeV compared to 7.3 MeV. Also a larger fraction of the deformed nuclei have very small values of the separation energy differences: 23% of the deformed nuclei have S_{n2p} less than $3.7/A^{2/3}$ MeV versus 9% for the other nuclei. The difference is not very large, suggesting that other mechanism beyond the recoupling effect may be needed. For example, configuration mixing arising from the neutron-proton interaction might depend on whether those nucleons are part of a pairing condensate or not. Such a mechanism would be beyond the usual DFT.

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